Cooling and Thermal Conduction in Clusters of Galaxies

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Abstract. We have been examining the role of heat conduction during the streams and other transitory event in the inner atmosphere of clusters of galaxies. Assuming the propagation of heat by thermal waves and hydrostatic equilibrium, we show that the conduction cooling timescale could be different from simple heat conduction. The implications for X-ray observations and interferometry are discussed.

1. Introduction

The thermal conductivity in the intracluster medium (ICM) is negligible owing to the low density; however, we may consider scenarios where thermal conduction and diffusion fronts are important, as in the appearance of violent temperature gradients as a consequence of processes of short duration (compared with the gas free-fall time) such as streams, supernova heating, super stellar winds, and AGN. These processes are usually invoked to explain the ICM metallicity. We believe that such scenarios could also recreate the conditions for the overcooling in the cluster atmosphere by heat waves.

Several authors (Zakamska & Narayan 2003; Dos Santos 2001; Henriksen & White 1996) have presented advances in the description of the hot gas in galaxy clusters in terms of hydrostatic equilibrium and thermal conduction. But in these models the heat propagation by waves is obviated by the assumption of the Maxwell–Fourier law for heat propagation: $\bar{F}(t + \tau, \vec{r}) \simeq \bar{F}(t, \vec{r}) = -k\vec{\nabla}T(t, \vec{r})$, a reasonable approach in the hot thin plasmas when the gradients of temperature are soft. If thermal pulses appear inside the plasma (SNe, AGN, streams), owing to high local temperature gradients the heat flux ($\bar{F}$) a given point, is the result of the instantaneous temperature gradient and also of the previous temperature gradients. Thus, $\bar{F}$ is the consequence of the thermal history of the medium and the relaxation time, $\tau$ (Jou, Casas-Vázquez, & Lebon 1999, and references therein):

$$\bar{F}(t, r) = -\int_{-\infty}^{t} \frac{k}{\tau} \vec{\nabla}T(t', r)e^{-(t'-t)/\tau} dt',$$

or, in differential form,

$$\tau \frac{d\bar{F}(r, t)}{dt} + \bar{F}(r, t) = -k\vec{\nabla}T(r, t),$$
where $k$ is the thermal conductivity. If $\tau \approx 0$ we recover the usual Maxwell–Fourier law. Following Zakamska & Narayan (2003), we consider a very simple gas model in hydrostatic equilibrium and thermal balance, with cooling exactly compensated by heat conduction, but using the Cattaneo law instead of the Maxwell–Fourier law and without dark matter.

2. The Model

We consider a spherically symmetric inner atmosphere cluster in equilibrium hydrostatic, where $(1/\rho) \nabla P = -\nabla \Phi$; the gas pressure dominates and the dynamical effect of the magnetic field is negligible. The distribution of the X-ray emitting plasma is governed by the cluster potential over timescales shorter than the dynamical time characteristic; i.e., the Poisson law is verified: $\nabla^2 \Phi = 4\pi G \rho$.

We adopt a polytropic temperature stratification, $T(r, t) \propto [n(r)]^{\gamma - 1}$. The polytropic exponent, $\gamma \in [1, 5/3]$, thus the smallest value ($\gamma = 1$) belongs to the isothermal assumption. Then

$$n(r) = n_c \left[ 1 + \left( \frac{\gamma - 1}{\gamma} \right) \frac{\Phi_c - \Phi}{k T_c / m} \right]^{\frac{1}{\gamma - 1}} \approx n_c \left[ 1 + \frac{r^2}{r_c^2} \right]^{-3/2}. \quad (3)$$

Obviously, this distribution of the number particle density (3) is resolved by construction the Poisson and hydrostatic equilibrium equations. The subscript “c” refers to the center values.

The time dependence of the temperature is provide by the energy balance, between radiative cooling and heating by thermal conduction in the diffusion approximation. This is

$$\nabla \cdot \vec{F}(t, r) = - \int_v \varepsilon(T, v) dv, \quad (4)$$

where $\varepsilon(T, v)$ is the emissivity in a neutral plasma at temperature $T$ and frequency $v$, given by (Sarazin 1988; Rybicki & Lightman 1979):

$$\varepsilon(T, v) = \frac{2^4 e^6 k_B}{(3m_e k_B / 2\pi)^{3/2} c^3} g_f n^2(r) T^{-1/2}(r, t) e^{-h v / k_B T}, \quad (5)$$

where $g_f$ is integrated Gaunt factor. But the radiative and diffusive flux are caused by equal temperature distribution and material distribution. Under this condition, then,

$$\frac{1}{r^2} \frac{d}{dr} [r^2 F(t, r)] = \frac{2^4 e^6 k_B^2}{(3m_e k_B / 2\pi)^{3/2} c^3 h} g_f n^2(r) T^{1/2}(r, t) \equiv \lambda_0 n^2(r) T^{1/2}(r, t). \quad (6)$$

Now, we are using the classical Spitzer thermal conductivity in fully ionized gases in (2), together with (6). With the assumption $T(r, t) = Tr(r) \cdot T(t)$, we now obtain the Bernoulli equation:

$$\frac{\tau}{2} \frac{dT_t}{dt} + T_t = -Q(r) T_t^4, \quad (7)$$
where
\[ Q(r) \approx \frac{40\sqrt{2}k_Bn^3/2}{37m_e^{1/2}\lambda_0e^{4\pi^3/2}} \frac{T_r^{5/2}dT_r}{dr} \left[ \frac{1}{r^2} \int r^2n^2(r)T_r^{1/2}dr \right]^{-1}. \] (8)

Also, we used the usual approach for the Coulomb logarithm that \( \ln \Lambda \approx 37. \)

We solve eq. 7 with initial condition \( T_t(0) = T_c. \) Together with the previous relation, \( T_t(t) = T_t(t)[n(r)]^{\gamma-1}, \) we obtain:

\[ T(t, r) = T_c e^{-2t/\tau} \left[ 1 + Q(r) T_c^3 \left( 1 - e^{-6t/\tau} \right) \right]^{-1/3} \left[ 1 + \frac{r^2}{\tau^2} \right]^{-3\beta/2(\gamma-1)}. \] (9)

If we take the divergence in the law of Cattaneo (2) and replace the flux through the conservation of the energy, we obtain a hyperbolic equation that indeed represents the propagation of heat for waves (Jou et al. 1999):

\[ \frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau} \frac{\partial T}{\partial t} = v^2 \nabla^2 T, \] (10)

where \( v, \) the velocity of the heat waves, is given in terms of the thermal conductivity, \( k, \) and the specific heat at volume unity \( c_v \) by \( v^2 = k/(\tau c_v). \) The relaxation time can now be evaluated in terms of the central temperature, number density, and refraction index by heat waves, \( \chi, \) as

\[ \tau \simeq 1.9 \times 10^5 \chi^2 \gamma^{-1} \left[ \frac{T}{10^8 \text{ K}} \right]^{5/2} \left[ \frac{n}{10^{-3} \text{ cm}^{-3}} \right]^{-1} \text{ yr}. \] (11)

Obviously, \( 1 < \chi \) is not exactly known, but the second sound velocity is less than \( \sqrt{3}. \) Cooling by heat waves shows that heat conduction may be relatively effective in a core of clusters and also for outer core-radius scales. The usual thermal conduction timescale is very long compared with previous estimates (Sarazin 1988).

As a specific example, consider the Abell 2142 cluster of galaxies, in which the thermal conduction timescale is \( 1.76 \times 10^8 \) yr using the MF law, but only \( 10^5 \) yr in the Cattaneo regime. Also, the average of the central temperature in Abell 2142 would be changed if the dynamical cooling effect is considered.

3. Discussion

Thermal conduction is most effective when heat waves exist in the inner cores of clusters and groups of galaxies; the temperatures determined from X-ray spectra could then increase the emissivity because of the conduction. We can also see this if we take the divergence in (1) and replace it in (6), although, according to the Leibning rule,

\[ \varepsilon(T) = \lambda_0n^2(r)T^{1/2}(r, t) = -\int_{-\infty}^{t} \frac{k}{\tau} \nabla^2 T(t', r)e^{-(t'-t)/\tau} dt'. \] (12)

The emissivity is thus the result of the sum over the entire temperature distribution instead of just the central temperature profile. X-ray spectra are used
for the mass calculation; the correction could lessen the total mass in clusters
and furthermore increase the baryonic density.

On other hand, \( L \propto n^2T^{1/2} \), but \( T \propto n^{(\gamma-1)} \), as we saw in (12), so the true
central temperature is \( T \propto L^2n^{(3-\gamma)} \). Also the clusters with merging between
galaxies, starbursts, streams, and other transitory phenomena with high temperature gradients (e.g., hot flows in the Chandra–XMM APM08279+5255 observation). Not taking into consideration the time dependence in the temperature function would result in overestimation of the central temperature and density. Notice that the gas temperature function is static only if the characteristic time is longer than the relaxation time (when \( t/\tau \gg 1 \)), so that \( T(t, r) \cong T_c n(r)^{\gamma-1} \).

It is possible that the cooling flow extends to a large radius, farther than \( r_{500} \), where the electron temperature is not sufficient for the X-ray emission, because the heat is propagated rapidly by waves.

The Sunyaev–Zel’dovich measurement might be poorly affected because the Comptonization parameter is the linear product between \( n \) and \( T \); it is easily seen that the SZ temperature decrease is only changed by a factor equal to the polytropic exponent.

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References

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