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White Dwarf Cooling by Means of Heat Waves

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Abstract. The standard white dwarf (WD) cooling theory is revised when the heat flux is propagate by thermal waves in degenerate material. Before the relaxation time, the luminosity function is dampening oscillation and the WD-age increase.

1. The Cattaneo Law and the Energy Transport Equation

In the WD core the degenerate electrons provide a high thermal conductivity. But in the upper atmospheres, the matter is less and less degenerate, and the impurities increase, therefore the thermal conductivities change monotonously. Several researchers (e.g. Kowalski & Saumon 2004) state that the WD cooling timescale contains uncertainties due to conductivity, the rôle of convection, initial chemical composition, microscope diffusion, and other effects in WD atmospheres. In the degenerate material, as present in the WD core, heat can be propagated by thermal waves. Therefore the energy transport equation and the luminosity in WD stars change. However, in standard cooling theory the possibility of heat propagation by waves is ignored. This simplification will be spurious in the degenerate material because, the relaxation time (the time required to establish the heat flux when a temperature gradient is switched on) is not negligible. Here we describe how the cooling time and the WD luminosity change if heat waves are taken into account. The temperature gradient in the stellar interior is given, in a good approximation, in terms of local values of opacity κ, density ρ and energy flux F by:

$$\frac{dT}{dr} = -\frac{3\kappa\rho F}{4acT^3} \tag{1}$$

This is just the Fourier-Maxwell law for energy flux due to thermal conductivity and/or radiative diffusion:

$$\vec{F}(\vec{x},t) = -k\vec{\nabla}T(\vec{x},t) \tag{2}$$

It is well known that the Fourier-Maxwell law leads to a parabolic equation for T, according to which the perturbations are propagated with infinite speed (Jou et al. 1999). The origin of the non-causal behavior found in Eq. (2) is the assumption that the energy flux appears at the same time as the temperature gradient is switched on. A heat flux equation leading to a hyperbolic equation (telegraph equation) is the Cattaneo law (Falcon 2001), which may be written as an integral over the history of the temperature gradient:

$$\vec{F}(\vec{x},t) = -\frac{k}{\tau}\int_{-\infty}^{t} \exp\left[-(t-t')/\tau\right] \cdot \vec{\nabla}T(\vec{x},t')dt' \tag{3}$$

The relaxation time (τ) is, in general, very small ($\sim 10^{-11}s$ for the phonon-electron interaction and $\sim 10^{-13}s$ for the phonon-phonon and free electron interaction, at room temperature). There are, however, situations where τ may not be negligible in degenerate material, for example in the laboratory $\sim 10^{-11}s$, for super fluid helium II at T= 1.2 K, and $\tau \sim 10^{2}s$ in neutron star interiors at T=10^6 K (Herrera & Falcon 1995). Notice that Eq. (3) may be written in terms of the total luminosity as:

$$L = \frac{4\pi Rk}{\tau}\int_{-\infty}^{t} DT \cdot \exp\left[-(t-t')/\tau\right] dt' \tag{4}$$

where we have used $\nabla T \approx DT/R \equiv (T_{Central} - T_{Surroundings})/R$. On the other hand the change in central temperature is given by the rate with which the thermal energy of the superficial layer changes and can be written as:

$$L = C_V\frac{dDT}{dt} \equiv C_V\left(\frac{dT_{central}}{dt} - \frac{dT_{surroundings}}{dt}\right) \approx C_V\frac{dT_{surroundings}}{dt} \tag{5}$$

where it has been supposed that the time during which the superficial layer radiate is much smaller than the Kelvin-Helmholtz time. Eqs. (4) and (5) then give:

$$\frac{dDT}{dt} = \frac{1}{\tau\tau_d}\left\{\int_{0}^{t} DT(t')\exp\left[-(t-t')/\tau\right] dt' + \chi\cdot\exp(-t/\tau)\right\} \tag{6}$$

where τ_d is the time-scale of thermal adjustment, in other words the time it takes for a thermal fluctuation to travel the WD radius, which is given as:

$$\tau_d \equiv \frac{k\rho^2 d^2 c_p}{16acT^3} \quad ; \quad \chi \equiv \int_{-\infty}^{0} DT(t')\exp(t'/\tau)dt' \approx \tau\cdot DT(0) \tag{7}$$

Taking Laplace Transformations of both sides and some elementary algebra, one obtains:

$$DT(t) = DT(0)\cdot exp(x)\left\{cos(wx) - \frac{sin(wx)}{w}\left[1 + \frac{2\chi}{\xi\cdot DT(0)}\right]\right\} \tag{8}$$

with

$$x \equiv t/2\tau \quad ; \quad \alpha \equiv \omega^2 + 1 \equiv \frac{4\tau}{\tau_d} \tag{9}$$

Then feeding back Eq. (6) into Eq. (5) we obtain:

$$L = \left[4\pi RkDT(0)\exp(-t/\tau_d)\right]\cdot f(x,\alpha) \equiv L_O\cdot f(x,\alpha) \tag{10}$$

where

$$f(x,\alpha) = \frac{1}{\alpha} \cdot \left[(4+\alpha) \cdot \cos(\omega x) - \frac{(4-\alpha)}{\omega} \cdot \sin(\omega x) \right] \exp\left[\frac{x}{2}(\alpha - 2) \right] \tag{11}$$

The last term in Eq. (10) defines the standard luminosity (L_O) in the Maxwell Fourier regime (when the relaxation time is negligible). This equation connects the standard luminosity and the "true" luminosity before thermal relaxation (in the presence of heat waves).

2. The Cooling Time of White Dwarfs

The influence and importance of the mixing-length theory of convection in the study and calculation of atmosphere models for WD have been broadly reported. The simple radiative model (simple cooling model) ignores convection completely, which is a serious omission. If the WD core is degenerate (or with super fluid helium layers) then the relaxation time cannot be negligible. In this case the WD contains layers with quasi periodic luminosity variation while the times are less than relaxation time. Using the convection theory for the energy transport one finds in the Cattaneo regime that the luminosity has the form of Eq. (10). We assume hydrostatic equilibrium and the Cattaneo law in the energy transport equation for the temperature, in terms of F or in terms of luminosity, as:

$$\frac{dT}{dr} = -\frac{3}{4ac} \cdot \frac{\kappa\rho}{T^3} \cdot \left(\tau \frac{\partial F}{\partial t} + F \right) \quad \text{or} \quad \frac{dT}{dP} = \frac{3\kappa}{64\pi\sigma} \cdot \frac{1}{GMT^3} \cdot \left(\tau \frac{\partial L}{\partial t} + L \right) \tag{12}$$

Notice that if $\tau \approx 0$ the relations (3) and (12) are the "classical" equation for the transport of energy in the stellar interior. The presence of a sub-envelope or surface convection zone can highly affect the WD cooling rate and its age, as will be seen now. According to Eq. (10) and Eq. (12) then:

$$T^{3-j} \cdot P^{-i} \frac{\partial T}{\partial P} = \frac{3\kappa_0}{64\pi\sigma} \left(L^{(d)}/GM \right) \cdot \left[\left(1 - \frac{\tau}{\tau_d} \right) \cdot f(x,\alpha) + \tau \frac{\partial f(x,\alpha)}{\partial t} \right] \tag{13}$$

Now we use the Kramers opacity, and to simplify matters, we assume a discontinuous transition from degeneracy to non-degeneracy at a certain point (subscript 0), yielding:

$$T_0 \cong \vartheta^{2/7} \left(L^{(d)}/L_S \right)^{2/5} (M/M_S)^{-2/5} \left[\left(1 - \frac{\tau}{\tau_d} \right) \cdot f(x,\alpha) + \tau \frac{\partial f(x,\alpha)}{\partial t} \right]^{2/7} \tag{14}$$

Where T_0 is written in term of the solar mass (M_S) and luminosity (L_S) we get:

$$\vartheta \equiv B c_1^{-3} \Re^{-2} (L_S/L^{(d)})(M/M_S) \tag{15}$$

The energy equation allows the luminosity to be written as:

$$-L \approx c_v \frac{\partial T_0}{\partial t} M \tag{16}$$

Feeding back Eq. (14) and Eq. (10) into Eq. (16), we obtain the life time or the cooling time (τ_V):

$$\int_0^{\tau_V} \quad 1 + \frac{\tau}{\tau_d} + \frac{\tau}{f(x,\alpha)} \cdot \frac{\partial f(x,\alpha)}{\partial t} \quad ^{-1} dt = \frac{2}{5} \quad \frac{M_S}{L_S} \quad c_v \vartheta T^{-\frac{5}{2}} \tag{17}$$

Obviously when $\tau \approx 0$ we obtain the very well known (Falcon 2001) WD cooling time relation ($\tau_V^{(d)}$) without heat waves. In general, if heat waves exist, then the cooling time increases. Indeed, the general solution of Eq. (17) is:

$$2A_1\tau_v + 4A_2\tau \cdot \ln \quad -\frac{6}{\omega(\alpha+6)} sin(\omega x) + \cos(\omega x) \quad = \tau_V^{(d)} \tag{18}$$

where A_1 and A_2 are functions of τ and τ_d.

$$A_1 \equiv \frac{\alpha^2 + 9\alpha + 8}{\alpha(\alpha^2 + 11\alpha + 24)} \quad ; \quad A_2 \equiv \frac{\alpha + 8}{\alpha(\alpha^2 + 11\alpha + 24)} \tag{19}$$

It is easy to see that the non-linear term in left part of Eq. (18) is restrained. Also notice that the α parameter, by definition, is always positive.

3. Conclusion

The existence of a layer of degenerate fluid in WD interiors (in which the relaxation time could be as long as hundreds seconds) facilitates the propagation of heat waves. Eq. (10) shows that the luminosity depends on the previous history of the temperature gradient and the envelope composition. This result opens new possibilities not foreseen in the recent studies of fast cooling of white dwarfs (Prada Moroni & Straniero 2003), based on the Maxwell-Fourier law. In the coolest WDs, where the core is crystallized, Eq. (18) suggests that the relaxation time increases and the propagation of heat waves must be considered in the later phases of WD cooling. Notice that the relaxation time is another uncertainty, but could be estimated through ZZ Ceti light curves (Falcon 2003). The ages of the coolest WDs are very sensitive to luminosity and this can change the results for the oldest stellar ages and the Galactic disk. In the special case of $\tau \approx \tau_d$, the same as $\alpha = 4$ (critical case), the cooling time $\tau_V \cong 2.8\tau_V^{(d)}$; indicate that the oldest WD, if the causal propagation of heat is considered, will be two or three times older than suggested by the usual cooling time, based on the Maxwell-Fourier Law.

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